Electricity & Magnetism

Francken practice session

2/Apr/'26

1 — dielectrics
2 — Biot–Savart
3 — Ampère
4 — Helmholtz
5 — dia-/paramagn. (linear)

§1 $\vec{E}$ in a material $= ?$

$+q$, $-q$, $d\vec{r}$ } dipole $\vec{p} = q\vec{d}$ } $V_{dip} = (\;)$

$\vec{E}^{dip}$, $\vec{B}^{dip}$

material

dipole in unit volume $\vec{P} = \vec{p}/Vol$ } $V = \int(\;)$

? given $\vec{P}$, find $\vec{E} = ?$

$$V = \underbrace{\frac{1}{4\pi\varepsilon_0}\oint \frac{\vec{P}\cdot\hat{n}}{r}\,da'}_{\text{pot of surface ch.}} + \underbrace{\frac{1}{4\pi\varepsilon_0}\int \frac{(-)\vec{\nabla}\cdot\vec{P}}{r}\,d\tau'}_{\text{pot of vol ch.}}$$

$\sigma_b = \vec{P}\cdot\hat{n}$ $\quad \rho_b = -\vec{\nabla}\cdot\vec{P}$

? given $\vec{E}^{ext}$ } only from
find $\vec{E}$ in material.

(I) $\vec{P} \propto \vec{E}$ [linear dielectric]

$\vec{P} = \varepsilon_0 \chi_e \vec{E}$

$\vec{D} := \varepsilon_0\vec{E} + \vec{P}$
$= \underbrace{\varepsilon_0(1+\chi_e)}_{=\varepsilon}\vec{E}$

$\oint \vec{D}\cdot d\vec{a} = Q_f^{enc}$

1. ignore material ⇝ Vacuum
2. calculate $\vec{D}$ (using Gauss)
3. $\vec{E} = (1/\varepsilon)\vec{D}$ net field in material

§2

spherical, cylindrical

$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2} \leftarrow \text{curly}$$

? $\vec{B} = ?$

wire(s) ? $\vec{B} = ?$

$\vec{F}_C = q\vec{E}$

$\vec{F}_L = q\,\vec{v}\times\vec{B}$

LEFT

RIGHT

§3 I^{enc} = all current passing through

$\vec{I}\cdot\hat{n}$

$= \int \vec{J}\cdot d\vec{a} = \int \vec{J}\cdot\hat{n}\,da$

$I^{enc} = I$ any $S : \partial S = $ Circle

$\oint_{circle} \vec{B}\cdot d\vec{l} = \mu_0 I^{enc}$

circle

$d\vec{l}$

$\vec{B}(s)$

$I^{enc} = \frac{I}{\pi a^2}\pi s^2$

$J = I/\text{area}$

$I^{enc} = Ka$

$\vec{B}$

$\vec{K}$

N wires per length

$I^{enc} = I$

$B(s>a) = \text{const} = B(\infty) = 0$

$\oint \vec{B}\cdot d\vec{l} = L B^{inside}(s)$

§4 Helmholtz

any field $\vec{F} = -\vec{\nabla}U + \vec{\nabla}\times\vec{S}$

$$U(\vec{r}) = \frac{1}{4\pi}\int \frac{\vec{\nabla}\cdot\vec{F}}{\mathscr{r}}\,d\tau'$$

$$\vec{S}(\vec{r}) = \frac{1}{4\pi}\int \frac{\vec{\nabla}\times\vec{F}}{\mathscr{r}}\,d\tau' \quad \text{curly}$$

$\vec{E} : \vec{\nabla}\times\vec{E} = 0 \Rightarrow \vec{S}_E \equiv 0$

$\vec{B} : \vec{\nabla}\cdot\vec{B} = 0 \Rightarrow U_B \equiv 0$

$\vec{E}$ conservative
$\vec{B}$ curl-only

for $\vec{B}$: $\vec{A}$ exists
$\vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{A} \parallel \vec{I}$

We have freedom
$\vec{A} \mapsto \vec{A} + \vec{\nabla}f$
$\vec{\nabla}\times\vec{\nabla}f \equiv 0$
$U \mapsto U + \text{const}$ gauge

§5

$\vec{a} = \int d\vec{a}$
$= (\text{Area})\hat{n}$

$\vec{m} = I\vec{a}$

2nd leading

? given $\vec{M}$ find $\vec{B}$

$\vec{K}_b = \vec{M}\times\hat{n}$

$\vec{J}_b = \vec{\nabla}\times\vec{M}$

① find K_b, J_b; ② calc $\vec{B}$ from Ampère

? given $\vec{B}^{ext}$, find $\vec{B}$ net

(I) $\vec{M} \propto \vec{B}$, $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

$\vec{B} = \mu\vec{H}$, $\vec{M} = \chi_m\vec{H}$

$\mu_0(1+\chi_m)$

χ_m