Dated: 25 Nov. 2025

COHOMOLOGY OF THE KONTSEVICH GRAPH COMPLEX

MARKUSS G. KENINS

Introduction. Poisson manifolds naturally describe classical mechanics. The semiclassical approximation is done by deforming the Poisson structure $\mathcal{P}$. In 1996/7, Kontsevich [1, 2] showed that any Poisson structure can be universally deformed to remain Poisson to first order.[1] The deformation is described using graphs, and the relevant formalism is Batalin-Vilkovisky (BV) [10–13].

In particular, the relevant graphs are non-trivial cocylces (elements of the cohomology) with respect to a differential $d = [\bullet\!\!-\!\!\bullet, \cdot]$ on the Kontsevich graph complex. This century has been devoted to studying this cohomology. A breakthrough came in 2010–15, when Willwacher established that the zeroeth comology is isomorphic to the Grothendieck–Teichmüller Lie algebra [14]. Using BV formalism, the space of deformations is at least countable [9, 15].

Recent work [16,17] has been devoted to computing higher-order cohomologies of graph subcomplexes and loop orders. Cohomology is not fully known in loop order ≥ 12. This allows characterisation of non-trivial cocylces, hence deformations.

Physical relevance. If every universal deformation constructed in this process is exact (in differential $\partial_{\mathcal{P}} = [\![\mathcal{P}, \cdot]\!]$), then every Poisson flow amounts to observing non-linear changes of coordinates [9]. This is a very hard open problem [9].

Objectives.

(1) Reproduce the theory of Kontsevich and Willwacher to understand graph cohomology and deformation quantisation.
(2) Compute (unknown at the time) cohomology for relevant graph subcomplexes and in higher loop order.
(3) Find infinite families in loop order for which k-th cohomology is trivial. Either prove, find (counter-)examples, or motivate by computations on many examples, forming conjectures.
(4) Explicitly characterise basis of cohomology (as above) – what are (independent) non-trivial cocyles? What are the corresponding deformations?

Methods. The construction of the Kontsevich graph complex is as a differential graded Lie algebra [5]. Standard tools for cohomology computations are part of a good algebraic topology course [18]: spectral sequences, stable operations, Cartan's theorem, and Steenrod squares in relation to homotopy groups, K-theory.

Recent computations [16] utilise a Feynman transform.

Background. I am familiar with Kontsevich graph calculus to the extent of [1, 5, 9]. I have taken an algebraic topology course (primarily on homology), and am reading [18]. I have read selected parts of Willwacher's papers [14, 16, 17, 19] referenced here. I am following a master's course on bi-vectors, bundles, and brackets by Dr Arthemy Kiselev, and am reading accompanying literature: [20].

[1]Further explanations in [3–8] and lectures [9].

At ETH Zürich I will take 'Algebraic Topology II', which is focused on cohomology, among other relevant courses (see motivation letter) and read classical geometry literature [21–26].

Timeline.

Weeks	
1–8	Reproduce the basic & foundational elements of the theory by Kontsevich, Willwacher, BV formalism, and selected papers by Kiselev – objective (1).
9–16	Reproduce recent results by Willwacher. Make computations and characterisations of cohomology – (2) and (3).
17–21	Finalise computations, characterise cocycles and deformations – (4).
22–28	Finish working. Write thesis, incorporating any papers (preprints) produced.

References

[1] Maxim Kontsevich. Formality conjecture. In D. Sternheimer, J. Rawnsley, and S. Gutt, editors, *Deformation theory and symplectic geometry (Ascona 1996)*, pages 139–156. Kluwer Academic Publishers, 1997.

[2] Maxim Kontsevich. Deformation quantization of Poisson manifolds. *Letters in Mathematical Physics*, 66(3):157–216, December 2003.

[3] Christine Jost. Globalizing L_∞-automorphisms of the Schouten algebra of polyvector fields. *Differential Geometry and its Applications*, 31(2):239–247, 2013.

[4] C. Jost. *Topics in Computational Algebraic Geometry and Deformation Quantization.* PhD thesis, Stockholm University, 2013.

[5] Nina J Rutten and Arthemy V Kiselev. The defining properties of the Kontsevich unoriented graph complex. *Journal of Physics: Conference Series*, 1194:012095, April 2019.

[6] Ricardo Buring, Arthemy V. Kiselev, and Nina J. Rutten. The heptagon-wheel cocycle in the Kontsevich graph complex. *Journal of Nonlinear Mathematical Physics*, 24(Supplement 1):157, 2021.

[7] Arthemy V. Kiselev and Ricardo Buring. The Kontsevich graph orientation morphism revisited. *Banach Center Publications*, 123:123–139, 2021.

[8] Ricardo Buring and Arthemy V Kiselev. The orientation morphism: from graph cocycles to deformations of Poisson structures. *Journal of Physics: Conference Series*, 1194:012017, April 2019.

[9] A. V. Kiselev. Deformations of Poisson brackets according to Kontsevich: a graph complex and a graph orientation morphism. Mini lecture course at the Independent University of Moscow; recordings (in English) available at `www.mathnet.ru/rus/conf1591`, May 2019.

[10] I.A. Batalin and G.A. Vilkovisky. Gauge algebra and quantization. *Physics Letters B*, 102(1):27–31, 1981.

[11] I. A. Batalin and G. A. Vilkovisky. Quantization of gauge theories with linearly dependent generators. *Phys. Rev. D*, 28:2567–2582, Nov 1983.

[12] Arthemy V Kiselev. The geometry of variations in Batalin-Vilkovisky formalism. *Journal of Physics: Conference Series*, 474:012024, November 2013.

[13] Arthemy V. Kiselev. The calculus of multivectors on noncommutative jet spaces. *Journal of Geometry and Physics*, 130:130–167, August 2018.

[14] Thomas Willwacher. M. Kontsevich's graph complex and the Grothendieck–Teichmüller Lie algebra. *Inventiones mathematicae*, 200(3):671–760, June 2014.

[15] Sergei Merkulov and Thomas Willwacher. Grothendieck–Teichmüller and Batalin–Vilkovisky. *Letters in Mathematical Physics*, 104(5):625–634, April 2014.

[16] Thomas Willwacher. On the bridgeless graph complex. Preprint, arXiv: 2503.19830, available at `https://arxiv.org/abs/2503.19830`, 2025.

[17] Thomas Willwacher. The 11-loop graph cohomology. Preprint, arXiv: 2508.13724, available at `https://arxiv.org/abs/2508.13724`, 2025.

[18] A. T. Fomenko and V. L. Gutenmacher. *Homotopical Topology.* Academic Publishing House, Budapest, 1986.

[19] Vasily Dolgushev, Christopher L. Rogers, and Thomas Willwacher. Kontsevich's graph complex, GRT, and the deformation complex of the sheaf of polyvector fields, 2015.

[20] S. P. Novikov and I. A. Taimanov. *Modern geometric structures and fields*, volume 71 of *Graduate studies in mathematics.* American Mathematical Society, 2006.

[21] B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov. *Modern geometry – methods and applications. Part I. The geometry of surfaces, transformation groups, and fields*, volume 93 of *Graduate texts in mathematics.* Springer-Verlag, 1984.

[22] B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov. *Modern geometry – methods and applications. Part II. The geometry and topology of manifolds*, volume 104 of *Graduate texts in mathematics.* Springer-Verlag, 1984.

[23] B. A. Dubrovin, A. T. Fomenko, and S. P. Novikov. *Modern geometry – methods and applications. Part III. Introduction to homology theory*, volume 124 of *Graduate texts in mathematics.* Springer-Verlag, 1984.

[24] M. Postnikov. *Lectures in Geometry. Semester III. Smooth manifolds.* Mir publishers Moscow, 1989.

[25] M. Postnikov. *Lectures in Geometry. Semester IV. Géoméetrie Diffeérentielle.* URSS Moscow, 1994.

[26] M. Postnikov. *Lectures in Geometry. Semester V. Lie Groups and Lie Algebras.* Mir publishers Moscow, 1986.